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CP violation in 5D split fermions scenario

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Abstract: We give a new configuration of split fermion positions in one extra dimension with two different Yukawa coupling strengths for up-type, h_u, and down-type, h_d, quarks at $\frac{h_u}{h_d} = 36.0$. The new configurations can give enough CP violating (CPV) phase for accommodating all currently observed CPV processes. Therefore, a 5D standard model with split fermions is viable. In addition to the standard CKM phase, new CPV sources involving Kaluza-Klein(KK) gauge bosons coupling which arise from the fact that unitary rotation which transforms weak eigenstates into their mass eigenstates only holds for the zero modes which are the SM fields and not for the KK excitations. We have examined the physics of kaon, neutron, and B/D mesons and found the most stringent bound on the size R of the extra dimension comes from $|\epsilon_K|$. Moreover, it depends sensitively on the width, σ, of the gaussian wavefunction in the extra dimension used to describe of the fermions. When $\sigma/R \ll 1$, the constraint will be lifted due to GIM suppression on the flavor changing neutral current(FCNC) and CPV couplings.

Keywords: Beyond Standard Model, Compactification and String Models, CP violation.

Contents

1. Introduction

Recently, the prospect of large extra dimensions has been introduced [1] as an alternative view of gauge hierarchy problem of the Standard Model(SM). The extra dimension scenario also opens up new possible avenues for exploring physics beyond SM and many new degrees of freedom can now be entertained. Following the same line a new geometrical interpretation of the observed fermion mass hierarchy was introduced in ref. [2] by assuming that SM chiral fermions are localized at different positions by a background potential in the extra dimensional space y. Depending on the details of the model gaussian or exponential wave functions in y are for these zero modes. The resulting effective 4-D Yukawa hierarchy can be viewed as the overlapping wave functions between two different chiral fermions and is exponentially suppressed as their relative distance Δy. The further apart two chiral fermions are the smaller their 4D Yukawa and hence a smaller mass for the fermion.

This setup will naturally generate the tree level flavor changing coupling of Kaluza-Klein(KK) gauge bosons, which can provide many interesting flavor changing neutral current effects. This has been studied in detail for the charged leptons in [3]. The model

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used is the 5D SM with gauge and Higgs bosons all propagating in the bulk whereas the fermions are localized in the fifth dimension with an unspecified potential. It is customary to assume the fermions all have a gaussian distribution in y, and one universal Yukawa coupling to the bulk Higgs field. A realistic configuration was found by [4] numerically that fits the observed quark masses and the quark mixing matrix V_{CKM}. Later on, it was pointed out by [5] that because the structure zeros in the quark mass matrices of this particular solution the resulting Jarlskog invariant [6] $J \equiv |\operatorname{Im}(V_{ub}V_{cs}V_{us}^*V_{cb}^*)| \sim 10^{-9}$ is four order of magnitude below the observed CP violation (CPV) effects. The authors in [5] went on and obtained solutions that accommodate the observed CPV phenomenon by extending to two extra dimensions.

In this paper we point out that a minimum extension of the 5DSM with different Yukawa couplings h_u for up-type and h_d for down-type quarks has a solution at $h_u/h_d = 36.0$ which reproduces observed mass spectrum and the values of the CKM mixing angles in the left handed charged current and at the same time yields the desired strength of CPV effects. Since the assumption of universal Yukawa coupling is made on account of simplicity we deemed it worthwhile to explore the effects of relaxing this. Furthermore, it is not a priori clear that a viable solution to quark masses and the CKM matrix can be found. Indeed an extensive numerical scan is required just as in the case of [2]. This is described in section 3.

The 5DSM is very rich in FCNC effects involving the KK excitations of the gauge and the Higgs bosons [9] and [3]. New CPV couplings also accompany these excitations. In addition to the one phase in V_{CKM} that operates on left-handed quarks there are phases related to the rotation of the right-handed quarks. The latter rotation is important since KK excitations of the gauge bosons except the W boson all couple to the right-handed quarks. The same is true for the KK Higgs bosons. Given that there are severe experimental limits on these effects nontrivial constraints are expected. It is our purpose to study in detail such constraints and how they impact the physics of this class of models. After going through all the consideration of current experimental limits on FCNC and CPV, it is found that the kaon CPV parameter $|\epsilon_K|$ gives the most stringent limit on R, the size of the compactified extra dimension. However, the limit on R strongly depends on the ratio of $\rho \equiv \sigma/R$ where σ is the gaussian width of the fermions which is basically a free parameter in this model. For $\rho = 10^{-2}$ we obtain $R^{-1} > 7 \times 10^3\ TeV$ whereas for $\rho = 10^{-6}$ no meaningful bound on R is found. This is due to the fact that as ρ becomes smaller the Glashow-Iliopoulos-Maiani(GIM) [7] mechanism is more operative and the FCNC and the accompanying CPV effects are suppressed as in the SM.

This paper is organized as follows: In section 2, we first outline the 5DSM model with split fermions. One bulk Higgs field is employed to give masses to the zero mode fermions. It has a constant profile in y. We are assuming that the localization of fermions is due either to addition bulk scalar fields with nontrivial profiles or some other yet unknown mechanism. The gauge boson are also bulk fields. Only the necessary pieces of the effective 4D lagrangian are given. For details of gauge fixing and the derivation of the Feynman rules we refer to ref. [3]. For simplicity a gaussian profile with a universal width σ is given to all the quarks. This is a free parameter of the model and is natural for consistency of the model

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to have $\sigma \ll R$. The fermion KK excitations in this scenario will be much heavier then those of the gauge and Higgs bosons level by level and we shall not investigate them here. In section 3 we describe the solution we found for the positions in y of the chiral quarks. They are then used to calculate the rotation matrices for the chiral quarks. This can be done up to some arbitrary phases. In section 4, we examine the FCNC and CPV constraint from kaon physics. We also calculate the decay of $K^0 \to \pi\nu\bar{\nu}$ in this model. In section 5, we show that the new physics discuss here will lead to violation of weak universality. The focus in section 6 is on how the new sources of universality violation and CPV in this model on free neutron decays. This is important for next generation of cold neutron experiments. The electric dipole moment of the neutron will also be discussed. Section 7 contains the conclusion. Finally the needed Feynman rules are collected in appendix B

2. 5D SM model with split quarks

In [3] a 5DSM on the orbifold S_1/Z_2 with localized split leptons and Higgs and gauge bosons propagating in the fifth dimension was constructed. The derivation of the effective 4D lagrangian on the orbifold fixed point from the 5D bulk lagrangian is given there and is easily extended to the quark sector. We make an additional assumption that the Yukawa couplings for the up type quarks ,h_u, and the down type quarks ,h_d, are different. After integrating out the fifth dimension and performing the usual Kaluza-Klein decomposition of the 5D fields with the appropriate orbifold boundary conditions, the 4-D effective interaction of the KK gauge boson with the zero mode mass eigenstate quarks can be summarized as follows:

$$\begin{aligned}
\frac{\mathcal{L}^{KK}}{\sqrt{2}} &= -g_s G_\mu^{(n)}\overline{q_i}\gamma^\mu T^a \left(U_L^{(n)}\hat{L} + U_R^{(n)}\hat{R}\right) q_j - \\
&\quad - eA_\mu^{(n)}\overline{q_i}\gamma^\mu \left(U_L^{(n)}\hat{L} + U_R^{(n)}\hat{R}\right) q_j - \\
&\quad - \left(\frac{g_2}{\cos\theta}\right) Z_\mu^{(n)}\overline{q_i}\gamma^\mu \left(g_L^q U_L^{(n)}\hat{L} + g_R^q U_R^{(n)}\hat{R}\right) q_j - \\
&\quad - \frac{g_2}{\sqrt{2}} W_\mu^{(n)}\overline{u_i}\gamma^\mu U_L^{(n)}\hat{L} d_j + \text{H.c.}
\end{aligned} \tag{2.1}$$

where the $G_\mu^{(n)}$, $A_\mu^{(n)}$, $Z_\mu^{(n)}$ and $W_\mu^{(n)}$ are the $n-$th KK excitations of the gluon, photon, Z boson, and W boson respectively. The chiral projection operators are $\hat{R} = \frac{1+\gamma_5}{2}$ and $\hat{L} = \frac{1-\gamma_5}{2}$ and the family index is $i = 1, 2, 3$. The rest are standard notations for the SM. The matrices $U_{L/R}^{(n)}$ are a combination of the unitary transformations $V_{L/R}$ that takes the weak quark eigenstates to their mass eigenstates and the cosine weighting of the n-th KK modes. They are results after integrating out gaussian distribution of localized fermions in the fifth dimension. Explicitly, we have

$$U_L^{(n)} = V_L^\dagger \begin{pmatrix} c_{n1}^L & 0 & 0 \\ 0 & c_{n2}^L & 0 \\ 0 & 0 & c_{n3}^L \end{pmatrix} V_L\,, \qquad U_R^{(n)} = V_R^\dagger \begin{pmatrix} c_{n1}^R & 0 & 0 \\ 0 & c_{n2}^R & 0 \\ 0 & 0 & c_{n3}^R \end{pmatrix} V_R \tag{2.2}$$

where the short hand notation $c_{ni}^{L/R} \equiv \cos(ny_i^{L/R}/R)$ and y_i is the fixed location in the fifth dimension of the quark q_i. Appendix B gives the detail structure of the U matrices in component forms.

These U matrices encode FCNC information since they are not diagonal in the quark mass basis. Clearly when the fermions are not split, i.e. all the $y_i = 0$ these U matrices become the identity matrix and the KK excitations will not have FCNC couplings. This is a general feature of the split fermion scenarios that gives rise to FCNC couplings for KK gauge boson to zero mode fermions. A second feature of phenomenological importance is the presence of U_R and now the KK gauge bosons are sensitive to rotations of the right-handed quarks. Now we have effective extra Z bosons without explicitly adding another gauge group. Similarly there are a host of extra Higgs bosons although we only have one Higgs doublet albeit it is a bulk filed. Furthermore, the U matrices also contain additional CPV phases which are distinct from the $V_{\rm CKM}$ phase. We shall see later that the positions y_i can be expressed in units of σ and some can be as far away as few tens of σ from the orbifold fixed point. It is reasonable to assume that ρ to be less than 10^{-2}. From eq. (2.2) the strength of FCNC and CPV interactions are controlled by ρ. For the first few KK states an expansion in ρ is accurate and the role of the GIM mechanism is evident. We shall defer the discussion of this phenomenology to later sections and instead turn our attention to the quark mass matrices.

The quark mass matrices stem from the interaction of fermions and the vacuum expectation value (VEV) of Higgs zero mode are given by

$$M^{U(0)}_{\{ij\}} = -\frac{v_0 h_{u\{ij\}}}{\sqrt{2}} \exp\left[-\frac{\Delta_{ij}^2}{4\sigma^2}\right], \qquad M^{D(0)}_{\{ij\}} = -\frac{v_0 h_{d\{ij\}}}{\sqrt{2}} \exp\left[-\frac{\Delta_{ij}^2}{4\sigma^2}\right], \tag{2.3}$$

where $\Delta_{ij} = |y_i - y_j|$, the distance between flavor i and j. They are diagonalized by biunitary rotations of matrices $V_{L/R}$

$$M^U_{\rm diag} = V_U^{L\dagger} M^U V_U^R, \qquad M^D_{\rm diag} = V_D^{L\dagger} M^D V_D^R. \tag{2.4}$$

The Yukawa couplings of the $n-$th KK Higgs, $n > 0$, can be expressed explicitly as

$$h^{(n)} = \frac{g_2}{M_W} V_L^\dagger M^{(n)} V_R \tag{2.5}$$

where the $M^{(n)}$ matrix is the convolutions of mass matrix and the weight of n-th KK excitation

$$M^{(n)}_{\{ij\}} = M_{\{ij\}} \cos\frac{n\overline{y_{ij}^{LR}}}{R} \tag{2.6}$$

where $\overline{y_{ij}^{LR}} = (y_i^L + y_j^R)/2$. For completeness the 4D interaction of the quarks with the KK Higgs boson is given by

$$-\mathcal{L}_Y^{KK} = h^{(n)}_{d\{ij\}} \overline{Q_i} H^{(n)} d_j^R + h^{(n)}_{u\{ij\}} \overline{Q_i} \tilde{H}^{(n)} u_j^R + \text{H.c.} \tag{2.7}$$

where $H^{(n)}$ is the n-th KK Higgs doublet, $\tilde{H}^{(n)} = i\sigma_2 H^{(n)*}$, Q_i is the SU(2) doublet quarks and u_i and d_i are the up- and down-type SU(2) singlet quarks respectively. The detail

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form of these effective Yukawa matrices is given in appendix B. It suffices to note that again FCNC and CPV interactions are present in the KK Higgs couplings and they flip the chiralities of the quarks.

3. A new solution

In [5], the authors analyzed the CP violating properties of the solution obtained by [4]. They concluded that the resulting CP violation is not enough to accommodate the observed experiments in K and B systems so a total two extra dimensions are needed. The solution given in [4] is obtained by assuming a universal Yukawa coupling strength: $|h^{(0)}_{u\{ij\}}| = |h^{(0)}_{d\{ij\}}| = 1.5$ which is chosen to be bigger than unity so as to accommodate a separation between the left-handed and right-handed quarks of the third family. It is also claimed that no other solution was found. We have independently checked that this indeed is the case. Here we would like to investigate whether a solution can be found by giving the Yukawa couplings a minimal flavor structure. Hence, we relax the assumption of universal Yukawa coupling assumption. The minimum extension one can imagine is to allow two different couplings for up- and down-type right handed quarks. As in ref. [4] a value of $h_u = 1.5$ is used and h_d is now allowed to float. The solution has to pass the requirement that the observed mass spectrum and the CKM mixing, which are summarized in appendix A, can be reproduced. Also the solution is required to accommodate the experimental CPV processes, namely the resulting value of the Jarlskog invariant is big enough.

The strategy for numerical searching is the following: we start from the solution given in [4], which appears to be robust, and then slightly vary h_d from the initial value, 1.5, and let the program meander around the initial configuration to find a set of new positions for the fermions which pass the two criteria mentioned above. Then the new position and h_d are used as initial condition for next iteration that h_d is further driven away from h_u. We found that the second criteria can be fulfilled only when h_u/h_d is larger then 33.0 and we cannot find any solution for $h_u/h_d > 40.0$ which pass the first requirement. As an example, we give one of the solutions at $h_u/h_d = 36.0$, namely $|h^u_{\{ij\}}| = 1.5$ and $|h^d_{\{ij\}}| = 0.0417$ which satisfies all the requirements and the averaged resulting Jarlskog invariant is 10^{-5}.

$$Q_i = \sigma \begin{pmatrix} 0.0 \\ 14.2349 \\ 8.20333 \end{pmatrix}, \qquad U_i = \sigma \begin{pmatrix} 6.13244 \\ 20.092 \\ 9.64483 \end{pmatrix}, \qquad D_i = \sigma \begin{pmatrix} 19.4523 \\ 5.15818 \\ 10.1992 \end{pmatrix}. \tag{3.1}$$

In the notation of ref. [2] the gaussian width there is $\mu^{-1} = \sqrt{2}\sigma$. The solution was shifted in y-direction a little bit to make every fermion position positive such that there is no conflict with the S_1/Z_2 compactification.

The corresponding mass matrices at the scale of m_t are

$$|M_U| \simeq \begin{pmatrix} .02056 & 0 & 0 \\ 0 & 0.04694 & 1.28438 \\ 85.226 & 0 & 148.112 \end{pmatrix} \text{GeV}\,, \tag{3.2}$$

$$|M_D| \simeq \begin{pmatrix} 0 & 0.0087 & 0 \\ 0.0027 & 0 & 0.1178 \\ 0 & 0.6909 & 2.553 \end{pmatrix} \text{GeV}\,. \tag{3.3}$$

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They give quarks masses (c.f. [4])

$$\begin{aligned} m_u(2\,\mathrm{GeV}) &= 2.40\,\mathrm{MeV}\,, & m_c(m_c) &= 1.39\,\mathrm{GeV} \\ m_d(2\,\mathrm{GeV}) &= 3.61\,\mathrm{MeV}\,, & m_b(m_b) &= 4.10\,\mathrm{GeV} \\ m_s(2\,\mathrm{GeV}) &= 60.20\,\mathrm{MeV}\,, & m_t(m_t) &= 170.9\,\mathrm{GeV}\,. \end{aligned} \tag{3.4}$$

The resulting CKM matrix is

$$|V_{CKM}| = |(V_U^L)^\dagger V_D^L| \simeq \begin{pmatrix} 0.9748 & 0.2232 & 0.0018 \\ 0.2230 & 0.9741 & 0.0364 \\ 0.0099 & 0.0351 & 0.9993 \end{pmatrix}. \tag{3.5}$$

It is instructive to compare eq. (3.3) with the solutions of [4]. Our M_D has the same structure whereas M_U is not diagonal. The small elements in the (23) and (31) positions allow us to accommodate a CKM phase. Unlike the SM, the coupling of gauge boson KK excitations in general can be flavor and CP violating. There are $N_f(N_f+1)/2 = 6$, N_f is the number of family, phases for each left-handed and right-handed rotation matrices V_L and V_R. One can use six quarks mass eigenstates to absorb $2N_f - 1 = 5$ phases. So in total there are seven physical complex phases remained. Therefore, there should be seven corresponding Jarlskog-like invariant can be constructed [8]. The six additional phases are associated with the rotation of the right-handed zero modes which interact with the KK gauge and Higgs bosons. Notice that the gauge group is not extended. For the SM part, namely with zero mode fermions couple to the gauge boson zero modes, the only CP violating source is in the one CKM phase. One can check that the resulting SM Jarlskog $J \equiv |\,\mathrm{Im}(V_{ub}V_{cs}V_{us}^* V_{cb}^*)| \sim 10^{-5}$ which is the right amount to produce the desired CP phenomenology.

Since we have no knowledge about where and how large the additional phases are, and neither do we know the size of the parameter ρ, a Monte Carlo survey of various combinations are performed. Then we use the very accurate experiments on kaon to give us the acceptable values. This will be discussed in detail the next section. However, we note here that certain combinations of the matrix elements of the U matrices, table 1, are very useful as they occur frequently in our calculations. We list them for $\rho = \{10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}\}$. The first four rows are related to $\Delta S = 2$ operators and the others are related to $\Delta S = 1$ ones. The numbers represent the maximum absolute values allowed. For randomly chosen phases, the real and the imaginary parts fluctuate between plus and minus maximum absolute value.

One can learn from the result of our numerical experiment that as the ratio ρ goes down, the off-diagonal couplings become smaller. This is to be expected because in the limit $\rho \to 0$ the matrices $diag\{c_1^{L/R}, c_2^{L/R}, c_3^{L/R}\} \to \mathbf{1}_{3\times 3}$ and the coupling matrices $U_{L/R}^{(n)}$ also reduce to a three by three identity matrix. Note that in general the $\Delta S = 2$ transition is smaller than $\Delta S = 1$ one due to the same reason. Since U controls the strengths of the off-diagonal transition they are suppressed by GIM mechanism as $\rho \ll 1$. This relaxes the lower bounds on $1/R$ that exists in the literature which range from few TeV to few tens of TeV because ρ is often taken to be a free but fixed parameter. Physically, if one uses a

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	$\rho = 10^{-2}$	$\rho = 10^{-4}$	$\rho = 10^{-6}$	$\rho = 10^{-8}$
$\lvert\sum_{n=1}(U_{sd}^{(n)L})^2/n^2\rvert$	2.4×10^{-2}	3.3×10^{-5}	4×10^{-13}	4×10^{-21}
$\lvert\sum_{n=1}(U_{sd}^{(n)R})^2/n^2\rvert$	1.9×10^{-2}	6.5×10^{-5}	9.7×10^{-13}	9.7×10^{-21}
$\lvert\sum_{n=1}(U_{sd}^{(n)L}U_{sd}^{(n)R})/n^2\rvert$	1.5×10^{-2}	4.6×10^{-5}	6.2×10^{-13}	6×10^{-21}
$\lvert\sum_{n=1}(y_{sd}^n)^2/n^2\rvert$	1.1×10^{-5}	1.1×10^{-8}	1.3×10^{-16}	1×10^{-24}
$\lvert\sum_{n=1}(U_{sd}^{(n)L}U_{uu}^{(n)L})/n^2\rvert$	9.9×10^{-2}	4.6×10^{-4}	4.9×10^{-8}	5×10^{-12}
$\lvert\sum_{n=1}(U_{sd}^{(n)L}U_{dd}^{(n)L})/n^2\rvert$	9.3×10^{-2}	4.5×10^{-4}	4.9×10^{-8}	5×10^{-12}
$\lvert\sum_{n=1}(U_{sd}^{(n)L}U_{uu}^{(n)R})/n^2\rvert$	2.2×10^{-3}	2.2×10^{-4}	4.9×10^{-8}	5×10^{-12}
$\lvert\sum_{n=1}(U_{sd}^{(n)L}U_{dd}^{(n)R})/n^2\rvert$	9.5×10^{-4}	2.4×10^{-4}	4.9×10^{-8}	5×10^{-12}
$\lvert\sum_{n=1}(U_{sd}^{(n)R}U_{uu}^{(n)L})/n^2\rvert$	9.3×10^{-2}	6.8×10^{-4}	7.6×10^{-8}	9×10^{-12}
$\lvert\sum_{n=1}(U_{sd}^{(n)R}U_{dd}^{(n)L})/n^2\rvert$	8.8×10^{-2}	6.7×10^{-4}	7.6×10^{-8}	9×10^{-12}
$\lvert\sum_{n=1}(U_{sd}^{(n)R}U_{uu}^{(n)R})/n^2\rvert$	2.3×10^{-3}	3.4×10^{-4}	7.6×10^{-8}	9×10^{-12}
$\lvert\sum_{n=1}(U_{sd}^{(n)R}U_{dd}^{(n)R})/n^2\rvert$	3.5×10^{-3}	3.7×10^{-4}	7.6×10^{-8}	9×10^{-12}
$\lvert\sum_{n=1}(y_{sd}^n y_{uu}^n)/n^2\rvert$	3.9×10^{-3}	2.5×10^{-5}	2.8×10^{-9}	3×10^{-13}
$\lvert\sum_{n=1}(y_{sd}^n y_{dd}^n)/n^2\rvert$	5.0×10^{-6}	1.0×10^{-7}	1.4×10^{-11}	1.4×10^{-15}

Table 1: Upper bounds of various summations.

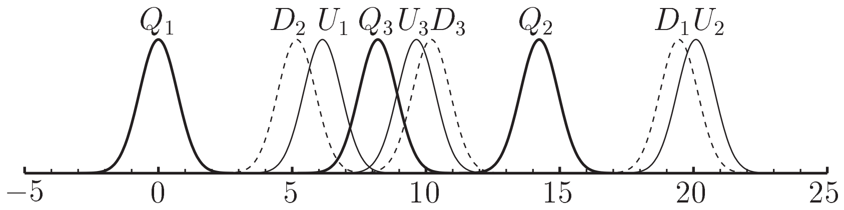

Figure 1: The map of fermions in the fifth dimension, in units of σ.

background potential to localize the fermions a small ρ corresponds to very a sharp kink. How to arrange for such a potential and its stability is beyond the scope of this paper. We note in passing that roughly speaking, the series sum can be expressed as ρ^k. Depending on the combination of flavor and chirality, the index k effectively varies from 1.5 to 3.2.

We give in figure 1 the geography of the quarks in the fifth dimension.

4. Kaon phenomenology

4.1 ΔM

At the tree level, the flavor changing $\Delta S = 2$ transitions can be mediated by exchanging neutral KK bosons, see figure 2. The effective $\Delta S = 2$ lagrangian can be read

$$\mathcal{L}_{\Delta S=2} = \sum_n \frac{2g_s^2}{3n^2/R^2}\left[U^n_{L\{ds\}}\bar{d}\gamma^\mu_L s + U^n_{R\{ds\}}\bar{d}\gamma^\mu_R s\right]^2 + \\ + \sum_n \frac{1}{n^2/R^2}\left(\frac{e}{3}\right)^2\left[U^n_{L\{ds\}}\bar{d}\gamma^\mu_L s + U^n_{R\{ds\}}\bar{d}\gamma^\mu_R s\right]^2 +$$

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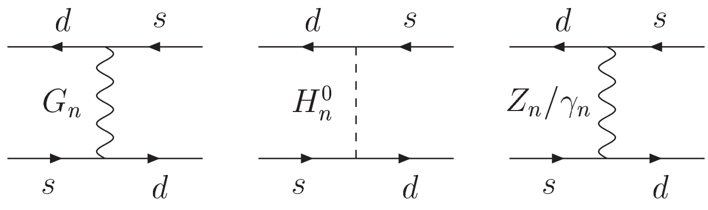

Figure 2: The dominant diagrams of KK exchange for ΔM.

$$+\sum_n \frac{1}{n^2/R^2}\left(\frac{g_2}{\cos\theta}\right)^2 \left[g_L U^n_{L\{ds\}}\bar{d}\gamma^\mu_L s + g_R U^n_{R\{ds\}}\bar{d}\gamma^\mu_R s\right]^2 +$$
$$+\sum_n \frac{1}{n^2/R^2}\left[h^n_{d\{ds\}}\bar{d}_R s_L + h^{n*}_{d\{ds\}}\bar{d}_L s_R\right]^2 . \tag{4.1}$$

Due to the larger coupling and color factors we can expect that the KK gluons dominate over the others and give a contribution beyond the SM as given by

$$\Delta m_K \sim 2\,\mathrm{Re}\,M_{12} \sim 2\,\mathrm{Re}\,M_{12}^{KKg}\,, \qquad M_{12} = \langle K_0|H_{\Delta S=2}|\bar{K^0}\rangle\,.$$

We using the standard method of vacuum insertion approximation(VIA) and the standard matrix elements [11],

$$\langle K_0|(\bar{s}\gamma^\mu_L d)(\bar{s}\gamma_{\mu L} d)|\bar{K^0}\rangle = \langle K_0|(\bar{s}\gamma^\mu_R d)(\bar{s}\gamma_{\mu R} d)|\bar{K^0}\rangle = \frac{1}{3} m_K f_K^2\,, \tag{4.2}$$

$$\langle K_0|(\bar{s}\gamma^\mu_L d)(\bar{s}\gamma_{\mu R} d)|\bar{K^0}\rangle = \left[\frac{1}{12}+\frac{1}{4}\left(\frac{m_K}{m_s+m_d}\right)^2\right] m_K f_K^2\,. \tag{4.3}$$

Plug in the solution we found, eq. (3.4), and use $m_K = 497.6\,\mathrm{MeV}$ [10] we have

$$\langle K_0|(\bar{s}\gamma^\mu_L d)(\bar{s}\gamma_{\mu R} d)|\bar{K^0}\rangle = 45.86\langle K_0|(\bar{s}\gamma^\mu_L d)(\bar{s}\gamma_{\mu L} d)|\bar{K^0}\rangle\,.$$

The chiral enhancement factor of 45.86 differs from [13] because our solution give a smaller current strange quark mass. However, it is easy to find an adjacent configuration which yields a bigger strange quark mass and given the uncertainty in the hadronic calculation the difference is not serious. Thus, we find the contribution of the KK gluons to the $K_L - K_S$ mass difference can be expressed as

$$\Delta m_K = -\frac{2g_s^2 R^2 m_K f_K^2}{9}\sum_{n=1}\frac{1}{n^2}\,\mathrm{Re}\left[(U^n_{L\{ds\}})^2 + (U^n_{R\{ds\}})^2 + 91.72\,U^n_{L\{ds\}}U^n_{R\{ds\}}\right], \tag{4.4}$$

or the limit on $1/R$ as

$$\frac{1}{R} > 1011\,\mathrm{TeV}\sqrt{\sum_{n=1}\frac{1}{n^2}\,\mathrm{Re}\left[(U^n_{L\{ds\}})^2 + (U^n_{R\{ds\}})^2 + 91.72\,U^n_{L\{ds\}}U^n_{R\{ds\}}\right]}\,, \tag{4.5}$$

where the numerical inputs used are: $\alpha_s \sim 0.1$, $f_K = 0.16\,\mathrm{GeV}$, $\Delta m_K = 3.488(79)\times 10^{-15}\,\mathrm{GeV}$ or $\Delta m_K/m_K \sim 7\times 10^{-15}$ [10].

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ρ	$\sqrt{L^2+R^2+2\times 45.86LR}$	$1/R$ (TeV)	
		Δm_K	$\lvert\epsilon_K\rvert$
10^{-2}	< 1.17	> 1200	> 14800
10^{-3}	< 0.37	> 370	> 4650
10^{-4}	$< 6.6\times 10^{-2}$	> 66	> 826
10^{-5}	$< 7.6\times 10^{-4}$	> 0.8	> 10

Table 2: Lower bounds of $1/R$ from Δm_K and $|\epsilon_K|$.

To a good approximation,

$$|\epsilon_K| \sim \frac{1}{2\sqrt{2}}\left|\frac{\mathrm{Im}\, M_{12}}{\mathrm{Re}\, M_{12}}\right|. \tag{4.6}$$

A similar expression from $|\epsilon_K| = 2.282\times 10^{-3}$ [10] gives constraint on $1/R$ as

$$\frac{1}{R} > 12579\,\mathrm{TeV}\sqrt{\sum_{n=1}\frac{1}{n^2}\,\mathrm{Im}\left[(U^n_{L\{ds\}})^2+(U^n_{R\{ds\}})^2+91.72U^n_{L\{ds\}}U^n_{R\{ds\}}\right]}. \tag{4.7}$$

We can put limits of $1/R$ by taking the maximum real or imaginary part as inputs, as listed in table 1 at various values of ρ and assume that the KK gluon accounts for most of the contribution to ϵ_K and obtain a stringent bound as seen in table 2. A caveat is in order. We expect the SM contribution for the CKM phase cannot be neglected. The relative size of the two contributions is not known. Hence, our estimate is an optimistic one. Moreover, the hadronic uncertainties are large and our bound is a good ball park number.

As an order of magnitude estimation, we can easily extend of the formulae for Δm_K to the cases of D and B mesons by simply replacing $f_K \to f_M$, $m_K \to m_M$ and $m_K/(m_s+m_d) \to m_M/(m_Q+m_q)$, where Q and q stand for the heavy and light quark respectively which constitute the meson M. Also the values of the U matrix elements are substituted by appropriate ones. Currently, these quantities are less accurately known as their kaon counter part and the constraint they impose are much looser as seen in table 3.

ρ	$1/R(\Delta m_D)$	$1/R(\Delta m_B)$
10^{-2}	> 34 TeV	> 3.9 TeV
10^{-3}	> 10.7 TeV	> 1.2 TeV
10^{-4}	> 2.0 TeV	> 0.3 TeV

Table 3: Bounds on $1/R$ from ΔM_D and ΔM_B.

For the $\Delta S = 1$ transition, the important quantity to consider is $|\frac{\epsilon'}{\epsilon}|$. In standard notation we have

$$\begin{aligned}\left|\frac{\epsilon'}{\epsilon}\right| &= \frac{\omega}{\sqrt{2}|\epsilon_K|}\left(\frac{\mathrm{Im}\, A_2}{\mathrm{Re}\, A_2}-\frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0}\right),\\ A_I e^{i\delta_I} &= \langle\pi\pi(I)|H_{\Delta S=1}|K^0\rangle\,,\end{aligned} \tag{4.8}$$

where $\omega = \mathrm{Re}\, A_2/\,\mathrm{Re}\, A_0 \sim 0.045$. Again, we expect that the most important contribution comes from KK gluon exchange. Following the notation of ref. [14], we find that the dominant contribution is from $Q_5 = (\bar{d}\gamma^\mu_L s)\sum_q(\bar{q}\gamma_{\mu R} q)$ and $Q_6 = (\bar{d}_i\gamma^\mu_L s_j)\sum_q(\bar{q}_j\gamma_{\mu R} q_i)$, the QCD penguin in the SM, for the $\Delta I = 1/2$ transition and $Q_7 = \frac{3}{2}(\bar{d}\gamma_L^\mu s)\sum_q e_q(\bar{q}\gamma_{\mu R} q)$, $Q_8 = \frac{3}{2}(\bar{d}_i\gamma_L^\mu s_j)\sum_q e_q(\bar{q}_j\gamma_{\mu R} q_i)$, the electroweak penguin in the SM, for the $\Delta I = 3/2$ transition. Here the indices i, j stand for color and also there are the operators $\tilde{Q}_i$ which are obtained from Q_i by the exchange $L \Leftrightarrow R$. Because parity is conserved in the strong interaction and assuming the weak phase is negligible, we have $\langle\pi\pi|Q_i|K^0\rangle = -\langle\pi\pi|\tilde{Q}_i|K^0\rangle$. The dominant $\Delta S = 1$ effective lagrangian relevant to Q_{5-8} due to KK gluon exchange is easy to compute and is given by

$$\begin{aligned}
\mathcal{L}^g_{\Delta S=1} &= \sum_{\substack{n=1\\ q=u,d}} \frac{g_s^2 R^2}{n^2}\Big[U^n_{R\{ds\}}U^n_{L\{qq\}}(\bar{d}\gamma^R_\mu T^a s)(\bar{q}\gamma^{\mu L}T^a q) + U^n_{L\{ds\}}U^n_{R\{qq\}}(\bar{d}\gamma^L_\mu T^a s)(\bar{q}\gamma^{\mu R}T^a q)\Big] \\
&= \sum_{\substack{n=1\\ q=u,d}} \frac{g_s^2 R^2}{n^2}\left[U^n_{R\{ds\}}U^n_{L\{qq\}}\left[\frac{1}{2}(\bar{d}_i\gamma^R_\mu s_j)(\bar{q}_j\gamma^{\mu L}q_i) - \frac{1}{6}(\bar{d}\gamma^R_\mu s)(\bar{q}\gamma^{\mu L}q)\right] + \right. \\
&\qquad \left. + U^n_{L\{ds\}}(U^n_{R\{qq\}}\left[\frac{1}{2}(\bar{d}_i\gamma^L_\mu s_j)(\bar{q}_j\gamma^{\mu R}q_i) - \frac{1}{6}(\bar{d}\gamma^L_\mu s)(\bar{q}\gamma^{\mu R}q)\right]\right]. \qquad (4.9)
\end{aligned}$$

Or we can rewrite it in the following form

$$\begin{aligned}
&\sum_{n=1}\frac{g_s^2 R^2}{6n^2}\left\{U^n_{L\{ds\}}\left[(U^n_{R\{uu\}}+2U^n_{R\{dd\}})\left(Q_6-\frac{1}{3}Q_5\right) + 2(U^n_{R\{uu\}}-U^n_{R\{dd\}})\left(Q_8-\frac{1}{3}Q_7\right)\right] + \right. \\
&\quad \left. + U^n_{R\{ds\}}\left[(U^n_{L\{uu\}}+2U^n_{L\{dd\}})\left(\tilde{Q}_6-\frac{1}{3}\tilde{Q}_5\right) + 2(U^n_{L\{uu\}}-U^n_{L\{dd\}})\left(\tilde{Q}_8-\frac{1}{3}\tilde{Q}_7\right)\right]\right\}, \qquad (4.10)
\end{aligned}$$

where the isospin symmetry breaking terms which are proportional to the difference of the coupling matrices of up and down quarks can be seen explicitly. Taking into account of the sign difference of $\langle Q_i\rangle$ and $\langle\tilde{Q}_i\rangle$, we computed the dominant contribution to be given by

$$\begin{aligned}
\left|\frac{\epsilon'}{\epsilon}\right| &\simeq \frac{\omega}{\sqrt{2}|\epsilon_K|\,\mathrm{Re}\,A_0}\times \\
&\times\left\{\sum_{n=1}\frac{g_s^2R^2}{6n^2}\,\mathrm{Im}\left[\left[U^n_{L\{ds\}}(U^n_{R\{uu\}}+2U^n_{R\{dd\}}) - U^n_{R\{ds\}}(U^n_{L\{uu\}}+2U^n_{L\{dd\}})\right]\times\right.\right. \\
&\times\left(\langle Q_6\rangle_0 - \frac{1}{3}\langle Q_5\rangle_0\right) - \frac{2}{\omega}\left[U^n_{L\{ds\}}(U^n_{R\{uu\}}-U^n_{R\{dd\}}) - U^n_{R\{ds\}}(U^n_{L\{uu\}}-U^n_{L\{dd\}})\right]\times \\
&\times\left.\left.\left(\langle Q_8\rangle_2 - \frac{1}{3}\langle Q_7\rangle_2\right)\right]\right\}. \qquad (4.11)
\end{aligned}$$

If we use the observed value of $|\epsilon'/\epsilon| = (1.8 \pm 0.4)\times 10^{-3}$ [10], the bound for R is very weak. Instead with the aid of VIA and the limits of $1/R$ derived from $|\epsilon_K|$, an upper bound on $|\epsilon'/\epsilon|$ from KK gluons can be predicted to be $\{3.4\times10^{-8}, 3.4\times10^{-8}, 2.3\times10^{-8}, 1.1\times10^{-6}\}$ at $\rho = \{10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$. Hence, for this model the direct CPV in kaon decays comes from the CKM phase and *not* the KK gluons or other gauge and Higgs excitations.

4.2 $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$

The two neutrinos semileptonic rare kaon decays are very sensitive probes of physics beyond the SM since they are relatively clean theoretically. Their discussion will involve the lepton configuration in y. Various kinds of constraint on the lepton positions in the extra dimension have been discuss [3]. Here our main purpose is to examine the contributions from the quark configurations we found. So we will take a simple set in which the lepton mass matrix is diagonal and use it as an example to evaluate the limit we can put on the size R of the new physics. This is sufficient for now. A complete job will require the knowledge of the values of the neutrino masses and the leptonic CKM matrix which are lacking. We are justified to neglect the question of neutrino mass in the extra dimension scenario. The interested reader can consult [12] and references therein. Without further ado we adopt the lepton positions in the fifth dimension given by [4] and shift them the same amount as we did for quarks:

$$L_i = \sigma \begin{pmatrix} 22.9943 \\ 8.7461 \\ 7.3319 \end{pmatrix}, \qquad E_i = \sigma \begin{pmatrix} 15.7429 \\ 14.3287 \\ 2.8774 \end{pmatrix}. \tag{4.12}$$

To a good approximation, the left-hand and right-hand rotation matrices for lepton can be treated as identity matrices. The main contribution now arise from KK Z boson exchange, see figure 3. Their couplings will simply be the SM coupling multiplied by the cosine weighting $\sqrt{2}\, c_n^{L/R}$ for the n-th KK boson.

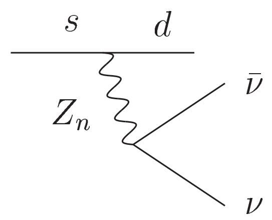

Figure 3: The dominant KK diagram for $K \to \pi\nu\nu$ decays.

We now have all the ingredients to compute the effective hamiltonian for the rare decay. It is given by

$$\begin{aligned} H_{\text{eff}} = & \frac{2g_2^2 g_L^s g_L^\nu}{\cos^2\theta_W} \sum_{\substack{n=1 \\ l=e,\mu,\tau}} \frac{U^n_{L\{sd\}} U^n_{L\{\nu_l\nu_l\}}}{n^2/R^2} (\bar{s}\gamma^\mu_L d)(\bar{\nu_l}\gamma_{L\mu}\nu_l) + \\ & + \frac{2g_2^2 g_R^s g_L^\nu}{\cos^2\theta_W} \sum_{\substack{n=1 \\ l=e,\mu,\tau}} \frac{U^n_{R\{sd\}} U^n_{L\{\nu_l\nu_l\}}}{n^2/R^2} (\bar{s}\gamma^\mu_R d)(\bar{\nu_l}\gamma_{L\mu}\nu_l) \end{aligned} \tag{4.13}$$

and we have adopted the notations of Buras [14] and extended the formulae to include new right-handed operators The branching ratio can be written in the following form

$$Br(K^+ \to \pi^+\nu\bar{\nu}) \simeq k_+ \left(\left| \frac{X_L}{\lambda^5} \right|^2 + \left| \frac{X_R}{\lambda^5} \right|^2 \right), \tag{4.14}$$

$$Br(K_L \to \pi^0\nu\bar{\nu}) \simeq k_L \left[\left(\frac{\operatorname{Im} X_L}{\lambda^5} \right)^2 + \left(\frac{\operatorname{Im} X_R}{\lambda^5} \right)^2 \right], \tag{4.15}$$

where

$$X_{L/R} = \frac{8\pi \sin^2\theta_W}{\alpha \cos^2\theta_W} \frac{g_L^\nu g_{L/R}^s R^2 M_W^2}{V_{ts}^* V_{td}} \sum_{n=1} \frac{U^n_{L/R\{sd\}} U^n_{L\{\nu\nu\}}}{n^2}. \tag{4.16}$$

The various quantities are $k_+ = 4.57 \times 10^{-11}$, $k_L = k_+\tau(K_L)/\tau(K^+) = 1.91 \times 10^{-10}$, and $\lambda = |V_{us}|$. Not surprisingly the FCNC couplings are analogous to those in the $|\epsilon_K|$ calculation. We can plug in the constraint on $1/R$ from before and get the following prediction: $Br(K^+ \to \pi^+\nu\bar{\nu}) < \{2.7 \times 10^{-17}, 3.3 \times 10^{-17}, 3.6 \times 10^{-17}, 3.7 \times 10^{-12}\}$ at

$\rho = \{10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$. This is to be compared to the SM prediction of $Br(K^+ \to \pi^+ \nu\bar{\nu})|_{SM} = (.75 \pm .29) \times 10^{-10}$ [15]. The result is very interesting compared to the present experiment value $Br(K^+ \to \pi^+ \nu\bar{\nu}) = (1.5^{+3.4}_{-1.2}) \times 10^{-10}$ [16]. X_L and X_R are essentially the amplitudes of the new physics. X_L is expected to interfere with the SM amplitude. Since we do not know the sign of the new phases relative to the CKM phase the numbers given only indicative and not robust numbers. As one can see, when $\rho = 10^{-5}$ the KK contribution can be as large as twenty percent of SM value at the amplitude level which will modify the branching ratio up to fifty percent. Even at $\rho = 0.01$, the branching ratio modification could also reach 0.1 percent level. It will be interesting to have more precise experimental bound. If the experimental result persists to be higher than the SM prediction it could signal a positive contribution from the mechanism we propose here.

A crude upper limit for $K_L \to \pi^0 \nu\bar{\nu}$ decay can be obtained by assuming the maximum allowed phases and simply multiplying a factor $k_L/k_+ = 4.17$ to the above prediction.

It will be difficult to obtain bounds or upper limits of the lepton flavor violation in K decays, for instance the processes $K^+ \to \pi^+ e^- \mu^+$, $K^+ e^+ \mu^-$, $K_L \to \mu e$ and $K_L \to \pi^0 e\mu$. Besides the strong dependence on the exact configuration of the lepton sector they also crucially depend on the leptonic CKM matrix which currently have no information. Hence, they are best left for future studies.

5. Universality of $\pi \to e\nu, \mu\nu$

The universality test of pion leptonic decays is a cornerstone for the SM. In this model, the interaction of the physical W is same as in the SM. So the universality tests using leptonic channels of physical W decay will not be altered from the SM prediction in this model [3]. However, the π decay will be modified by exchanging the virtual KK excitations as depicted in figure 4. Since the KK Higgs couplings to the fermions are suppressed by the light lepton masses we expect that the KK W exchange gives the dominant contribution. The resulting modification of decay rate ratio must satisfy the current universality test bound [10], i.e.

$$\Delta \left(\frac{\Gamma(\pi \to e\nu)}{\Gamma(\pi \to \mu\nu)} \right)_{KK} \sim M_W^2 R^2 \sum_{n=1} \frac{1}{n^2} \mathrm{Re} \left[U_{ud}^{L(n)} \left(U_{ee}^{L(n)*} - U_{\mu\mu}^{L(n)*} \right) \right] \lesssim 10^{-7} \,. \tag{5.1}$$

This is easily satisfied by the constraint on R and ρ derived from $|\epsilon_K|$. The universality violation will only be changed by the amount of $\{10^{-11}, 10^{-11}, 3 \times 10^{-11}, 2 \times 10^{-9}\}$ for $\rho = \{10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$.

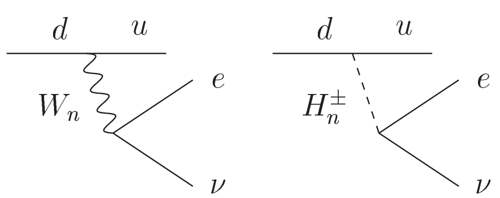

Figure 4: Dominant KK contributions for neutron beta decay and $\pi^\pm$ decays.

6. Neutron β decay

Following the notation of [17], the most general differential rate of a free neutron decays into proton plus electron and neutrino can be expressed as

$$
\begin{aligned}
d^2\Gamma \propto\ & E_e|\overrightarrow{p_e}|(E_e^{\max}-E_e)^2 dE_e d\Omega_e d\Omega_\nu \times \\
& \times\left[1+a\frac{\overrightarrow{p_e}\cdot\overrightarrow{p_\nu}}{E_eE_\nu}+b\frac{m_e}{E_e}+A\frac{P\cdot\overrightarrow{p_e}}{E_e}+B\frac{P\cdot\overrightarrow{p_\nu}}{E_\nu}+D\frac{P\cdot(\overrightarrow{p_e}\times\overrightarrow{p_\nu})}{E_eE_\nu}\right],
\end{aligned}
\tag{6.1}
$$

where P is the polarization vector of neutron. In the SM, neutron beta decay is solely mediated by virtual W boson exchanging. Neglecting the recoil correction, the coefficients are also given by [17]

$$
a=\frac{1-\lambda^2}{1+3\lambda^2}\,,\qquad b=0\,,\qquad A=2\frac{\lambda(1-\lambda)}{1+3\lambda^2}\,,\qquad B=2\frac{\lambda(1+\lambda)}{1+3\lambda^2}\,,
\tag{6.2}
$$

with $\lambda\equiv|g_A|/|g_V|$ and the CP violating triple correlation coefficient D is small. In principle, λ can be calculated by lattice QCD from the first principle and it has been precisely measured as 1.267(3) [10] which agree well with the measurements on the coefficients a, A, B.

In addition to the exchange of SM W boson, in the split fermion scenarios the neutron beta decay also receive contribution of KK W and KK charged Higgs boson, see figure 4. The new four Fermi interaction term is

$$
\begin{aligned}
H^\beta_{\rm int} &= \sum_{n=1}\frac{g_2^2}{2n^2/R^2}U^n_{L\{du\}}U^n_{L\{e\nu\}}(\bar u\gamma_{L\mu}d)(\bar e\gamma_L^\mu\nu) \\
&\quad+\sum_{n=1}\frac{1}{n^2/R^2}h^{n+}_{\{e\nu\}}\bar u[h^{n+}_{S\{ud\}}-\gamma^5h^{n+}_{P\{ud\}}]d[\bar{e_R}\nu_L]+\text{H.c.} \qquad (6.3)\\
&\equiv\frac{g_2^2V_{ud}}{2M_W^2}\left[a_W(\bar u\gamma_{L\mu}d)(\bar e\gamma_L^\mu\nu)+a_S(\bar ud)(\bar e\nu)+a_P(\bar u\gamma^5d)(\bar e\nu)\right]+\text{H.c.}\,, \qquad (6.4)
\end{aligned}
$$

where h^{n+}s are the coupling of fermions and KK charged Higgs bosons (refer to appendix B for their couplings). In general, a_W, a_S and a_P are complex. Now it is interesting to examine how the new interaction will affect the neutron decay, especially there are new CPV source in the KK W couplings and the presence of additional scalar interaction.

First, we note that the present of a_W and a_H will alter the total decay rate of neutron or the definition of G_β which will be discussed in the next subsection on $\Delta r_\beta-\Delta r_\mu$. We now look at effect of a_W more closely. At first glance it appears that there is a contribution to the CPV coefficient D. But from [17]

$$
\begin{aligned}
D &\simeq \frac{\xi}{2}\,{\rm Im}\left(\frac{{\rm Im}\,g_V}{g_V}-\frac{{\rm Im}\,g_A}{g_A}\right), \\
\xi &= 4\delta_{J_n,J_p}\left(\frac{J_n}{J_n+1}\right)^{1/2}\frac{\lambda M_FM_{GT}}{(1+|a_S|^2)|M_F|^2+\lambda^2|M_{GT}|^2}\,,
\end{aligned}
\tag{6.5}
$$

where M_F and M_{GT} are the Fermi and Gamow-Teller nuclear matrix elements, for neutron beta decay $|M_F|=1, |M_{GT}|=\sqrt{3}$. And J_n, J_p are the spin of neutron and proton respectively. Since KK charged current gives the same contribution to $g_V = g_V^{SM}(1 + a_W)$ and $g_A = g_A^{SM}(1 + a_W)$ at the quark level, unless the CPV be generated through long distance physics, we conclude there is no new effect on the CPV coefficient D in neutron beta decay. Also because the absence of tensor coupling at the tree level, the coefficients A and B will not be altered. The new scalar interaction will modify the a and b parameters and are given by

$$\begin{aligned} a &= \frac{(1-|a_S|^2)|M_F|^2 - \lambda^2|M_{GT}|^2/3}{(1+|a_S|^2)|M_F|^2 + \lambda^2|M_{GT}|^2}\,, \\ b &= \frac{2|M_F|^2\,\mathrm{Re}(a_S)}{(1+|a_S|^2)|M_F|^2 + \lambda^2|M_{GT}|^2}\,, \end{aligned} \tag{6.6}$$

with the current experimental data $a = -0.102(5)$ [10], a_S is bounded by $|a_S|^2 < 0.138$ and related to b as

$$a_S = \frac{\sqrt{2}R^2}{4G_F V_{ud}} \sum_{n=1} \frac{h^{n+}_{S\{ud\}} h^{n+}_{\{e\nu\}}}{n^2}\,, \qquad b = \frac{2\,\mathrm{Re}(a_S)}{5.82 + |a_S|^2}\,. \tag{6.7}$$

One immediately see that independent of ρ, b has to be in the range of ± 0.124. The prediction of a_S is very small, $< 10^{-14}$, due to the suppressed of electron mass. There is no hope to constraint this model by precision test of neutron beta decay.

6.1 $\Delta r_\beta - \Delta r_\mu$

It is well known that many new physics will modify the effective four Fermi coupling [3] and [18]. This holds true for the split fermions scenario in general, see figure 5 for the Feynman diagrams. We define the process dependent Fermi constant G_μ as

$$\begin{aligned} \frac{G_\mu}{\sqrt{2}} &\equiv \frac{g_2^2}{8M_W^2}\left[1 + \Delta r_\mu\right] \\ &= \frac{g_2^2}{8M_W^2}\left[\left(1 + R^2 M_W^2 (a_1 + a_3)\right)^2 + \left(\frac{a_2 + 2a_4}{2}\right)^2 R^4 M_W^4\right]^{\frac{1}{2}}. \end{aligned} \tag{6.8}$$

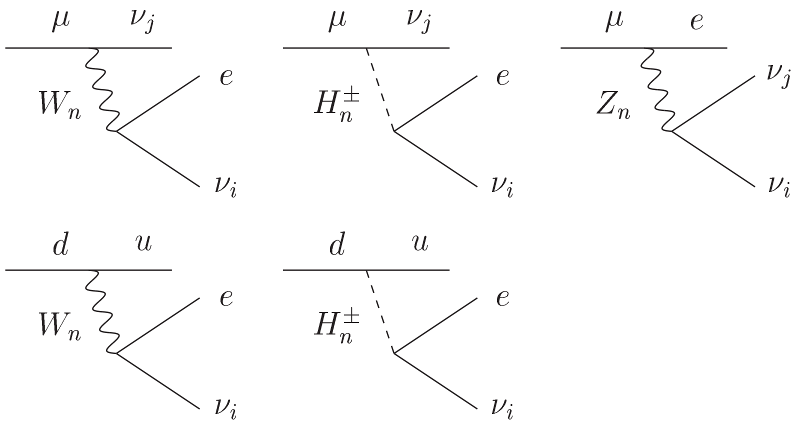

Figure 5: Tree level KK contributions to $\Delta r_\beta - \Delta r_\mu$.

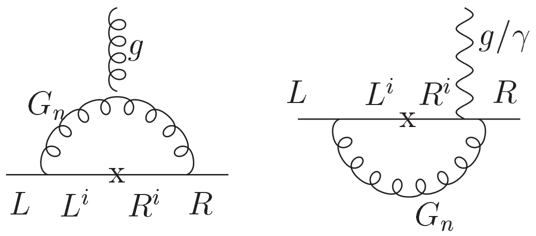

Figure 6: One loop diagrams for the chromo-dipole moment of a quark.

where the coefficients $a_{i,j,k}$ are the result of summing over all neutrino species and explicitly given by

$$a_1 \sim \sum_{i,j,n=1} \frac{U^n_{L\{j\mu\}} U^{n*}_{L\{ei\}}}{n^2}\,, \qquad a_2 \sim \sum_{i,j,n=1} \frac{4h^n_{\{j\mu\}} h^{n*}_{\{ie\}}}{g_2^2 n^2}\,,$$
$$a_3 \sim \left(\frac{2g^\nu_L g^e_L}{\cos^2\theta}\right) \sum_{i,j,n=1} \frac{U^n_{L\{e\mu\}} U^{n*}_{L\{ij\}}}{n^2}\,, \qquad a_4 \sim \left(\frac{2g^\nu_L g^e_R}{\cos^2\theta}\right) \sum_{i,j,n=1} \frac{U^n_{R\{e\mu\}} U^{n*}_{L\{ij\}}}{n^2}\,. \tag{6.9}$$

The square bracket in eq. (6.8) gives the modification to the SM Fermi coupling constant, $G_{SM,F} = \sqrt{2}\, g_2^2/8M_W^2$ and also generalizes the usual KK result [18]. We have a similar result for neutron beta decay:

$$\frac{G_\beta}{\sqrt{2}} = \frac{g_2^2 V_{ud}}{8M_W^2} \left[\left(1 + R^2 M_W^2 a_1^n\right)^2 + \left(\frac{a_2^n}{2}\right)^2 R^4 M_W^4 \right]^{\frac{1}{2}}, \tag{6.10}$$

where

$$a_1^n \sim \sum_{i,n=1} \frac{U^n_{L\{ud\}} U^{n*}_{L\{ei\}}}{V_{ud} n^2}\,, \qquad a_2^n \sim \sum_{i,n=1} \frac{4h^n_{\{ud\}} h^{n*}_{\{ie\}}}{V_{ud} g_2^2 n^2}\,.$$

They are due to the KK W and $H^\pm$ exchange. Since the new physics modifies the effective Fermi couplings differently and this is expressed in the following form

$$G_F^\beta = G_\mu V_{ud} (1 - \Delta r_\mu + \Delta r_\beta)\,. \tag{6.11}$$

Taking the previous diagonal lepton configuration, namely no FCNC in lepton sector, and expand the above formulas up to $\mathcal{O}(M_W^2 R^2)$,

$$(\Delta r_\mu - \Delta r_\beta)_{KK} \sim 2R^2 M_W^2 (a_1^\mu - a_1^n)\,. \tag{6.12}$$

With the inputs from kaon decays we predict the upper limits of $(\Delta r_\mu - \Delta r_\beta)_{KK} < \{6 \times 10^{-12}, 8 \times 10^{-11}, 2 \times 10^{-9}, 2 \times 10^{-5}\}$ for $\rho = \{10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$. Hence, violation of universality in these channels are expected to be small.

6.2 Neutron EDM

The neutron EDM is one of the most stringent test of CPV in the flavor conserving sector. Using the SU(6) quark model, the neutron EDM can be related to related to u(d) quarks'

EDM $d_u(d_d)$ and their chromoelectric dipole moments $d_u^g(d_d^g)$ as follows

$$\left(\frac{D_n}{e}\right) = \frac{4}{3}\left(\frac{d_d^\gamma}{e}\right) - \frac{1}{3}\left(\frac{d_u^\gamma}{e}\right) + \frac{4}{9}\left(\frac{d_d^g}{g_s}\right) + \frac{1}{9}\left(\frac{d_u^g}{g_s}\right). \tag{6.13}$$

As an estimate of the size of the new effects we assume that the chromoelectric dipole moment to be the dominating contribution to neutron EDM, see figure 6. We can ignore the Weinberg three gluon operator for now. Without going into the details of the calculation the one-loop transition amplitude can be estimated to be:

$$\begin{aligned}\left(\frac{d_u^g}{g_s}\right) &\sim \sum_{\substack{n=1\\ q=c,t}} \frac{g_s^2 m_q}{4\pi^2 M_n^2}\,\mathrm{Im}(U^n_{R\{qu\}}U^n_{L\{qu\}})\,,\\ \left(\frac{d_d^g}{g_s}\right) &\sim \sum_{\substack{n=1\\ q=s,b}} \frac{g_s^2 m_q}{4\pi^2 M_n^2}\,\mathrm{Im}(U^n_{R\{dq\}}U^n_{L\{dq\}})\,.\end{aligned} \tag{6.14}$$

To make a dipole operator, one need at least one mass insertion either on the internal or external fermion line to flip the chirality. Note that the light quark, u, d, contribution are absent because the flavor diagonal coupling is real. This is easy to see by looking at the component form of the coupling matrix, the complex numbers come in conjugate pair and the KK weighting factors $c_n^{L/R}$ are also real.

The contribution from the third family is enhanced by their masses but suppressed by the smallness of off-diagonal coupling between the first and third family. The factor $1/4\pi^2$ is due to the loop factor and $M_n^2 = n^2/R^2$ is from the propagator of KK gluon. By using the eq. (6.14) and the lower bound obtained from $|\epsilon_K|$, the predicted limits of neutron EDM are $d_n < \{6.6\times10^{-33}, 1.6\times10^{-33}, 2.6\times10^{-35}\}$(e-cm) for $\rho = \{10^{-2}, 10^{-3}, 10^{-4}\}$. Compared with the current experimental bound, $|d_n| < 6.3\times10^{-26}$e-cm [19]. This is even smaller then the expected SM contribution.

7. Conclusion

We have succeeded in finding a realistic 5D split fermion model,that yields the observed masses spectrum and give correct amount of CKM CPV phase. This is important since KK excitations alone will not account for ϵ'/ϵ. We achieved this by bestowing different Yukawa couplings for the up quarks, h_u, and for the down-quarks, h_d. We have fixed h_u at the value of 1.5 and vary h_d. Our numerical search indicates that the above requirements can be fulfilled only in a nontrivial and narrow window of h_d, $33.0 < h_u/h_d < 40.0$. What we presented is an existence proof but not necessary a unique solution. It also shows a moderate amount of tuning in the flavor structure but not excessive.

There are interesting new CPV and FCNC phenomenology in this model, i.e. the CPV and FCNC coupling of KK-excitations and SM fermions can be generated naturally at the tree level due to the fact that in the effective 4D theory the Yukawa matrix of KK modes receive different weighting from the SM mass matrix. The CPV characteristic of this model is unique in that there are CPV interactions in the neutral current sector that are of the $(V+A)$ and $(V-A)$ type. They arise from KK photons, KK Zs, and KK gluons.

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Observable	Constraint	Dominate KK
Δm_K	$\mathrm{Re}[U^2_{L\{sd\}} + U^2_{R\{sd\}} + 91.72 U_{L\{sd\}} U_{R\{sd\}}]$	G_n
$\lvert\epsilon_K\rvert$	$\mathrm{Im}[U^2_{L\{sd\}} + U^2_{R\{sd\}} + 91.72 U_{L\{sd\}} U_{R\{sd\}}]$	G_n
$\lvert\epsilon'/\epsilon\rvert$	$\mathrm{Im}[U_{L/R\{sd\}} U_{R/L\{qq\}}]$	G_n
$K^+ \to \pi^+ \bar{\nu}\nu$	$\lvert U_{L\{sd\}} U_{L\{\nu\nu\}}\rvert^2 + \lvert U_{R\{sd\}} U_{L\{\nu\nu\}}\rvert^2$	Z_n
$K_L \to \pi^0 \bar{\nu}\nu$	$\mathrm{Im}[U_{L\{sd\}} U_{L\{\nu\nu\}}]^2 + \mathrm{Im}[U_{R\{sd\}} U_{L\{\nu\nu\}}]^2$	Z_n
Δm_D	$\mathrm{Re}[U^2_{L\{cu\}} + U^2_{R\{cu\}} + 5.5 U_{L\{cu\}} U_{R\{cu\}}]$	G_n
Δm_B	$\mathrm{Re}[U^2_{L\{bd\}} + U^2_{R\{bd\}} + 5.2 U_{L\{bd\}} U_{R\{bd\}}]$	G_n
$\frac{Br(\pi\to\nu e)}{Br(\pi\to\mu\nu)}$	$\mathrm{Re}[U_{L\{ud\}}(U_{L\{ee\}} - U_{L\{\mu\mu\}})]$	$W_n^\pm$
$n \to p e \bar{\nu}$	$h^\pm_{S\{ud\}} h^\pm_{e\nu}$	$H_n^\pm$
$\Delta r_\beta - \Delta r_\mu$	$\lvert U_{L\{ud\}} U_{L\{e\nu\}}\rvert^2 - \lvert U_{L\{\mu\nu\}} U_{L\{e\nu\}}\rvert^2$	$W_n^\pm$
d_n	$\mathrm{Im}[\sum_{q=c,t} m_q U_{R\{qu\}} U_{L\{qu\}}] + 4\,\mathrm{Im}[\sum_{q=s,b} m_q U_{R\{qd\}} U_{L\{qd\}}]$	G_n

Table 4: Summary of constraint from phenomenology. It is understood the given expression actually is the sum over all contribution from KK excitation. Refer to the text for the accurate formulae.

Additional CPV coupling for $(V-A)$ charged current from KK W and S-type and P-type couplings from the KK neutral and charged Higgs bosons are present. There are No $(V+A)$ vector charged current in the model. Unlike the SM CKM phase, it has six extra complex phases arising from rotations of the SU(2) singlet right-handed quarks. In principle they are measurable once KK excitations are found. Obviously, to determine all of them we need at least seven linearly independent experiments to ping down all the phases.

To date, CPV effects are only observed in the Kaon and B system. In this scenario both the CKM phase and new physics can come into play. The relative amount of the two contributions are not known. We made the assumption that the new physics is at least as large as the SM in Δm_K and $|\epsilon_K|$. If the KK gluon contributions are only a fraction of the SM then the constraint on R is even stronger provided no fortuitous cancellations among elements of U.

From our discussions one can see that the most stringent constraint on R the extra dimension size comes from the kaon CPV parameter $|\epsilon_K|$. Depending on ρ the conservative lower bound on $1/R$ ranges from 10^4 TeV at $\rho = 10^{-2}$ to 10 TeV at $\rho = 10^{-5}$. If by accident the imaginary parts of the contribution from summing over KK excitation is nearly vanish, the real part, from Δm_K, will still give a strong constraint although one order of magnitude weaker. In table 4 we summarize the constraints from other decays we have examined. The rare decays of the kaon is next most sensitive and they probe a different source, i.e. the KK Z and thus are very complementary.

The implications of our results to collider physics depends crucially on ρ. The value of $\rho = 10^{-2}$ is not unnatural but lead to KK excitations in the mass range of 10^3 to 10^4 TeV which is out of reach for the foreseeable future. On the other hand if $\rho < 10^{-6}$ then TeV KK gauge bosons production can be expected. Moreover, the KK fermion will be completely out of reach since we expect the masses to be govern by $1/\sigma$. This is contrast to models which has all SM particles in the bulk [20].

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To sum up, we have shown that it is still viable to have a 5D split fermion model to explain both the flavor and hierarchy problem. However, extra dimension with size larger then $(\text{TeV})^{-1}$ will require a dynamical model which confines the fermions in a very small region in the extra dimension. The rich CPV and FCNC phenomenology of the model makes it interesting to pursue new rare kaon decay experiments as well as B and D decays since they probe different sectors of the model.

Although we have focused on one specific model with gaussian wavefunctions the feature of the tree-level FCNC and CPV couplings in KK sector is generic for this class of models with nontrivial fermion profiles. This is true also for models built with multi-located fermions(or branes) [21] regardless of the number of extra dimensions and the exact shape of how they spread in the extra dimension(s). Our analysis can be carried out in these models and similar tight constraints on these models are expected.

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A. SM parameters

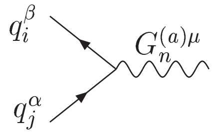
In the search of new configuration, we used the following allowed values of SM [10] quark masses at the scale of m_t, where the QCD and QED RG running [22] have been taken care of:

$$\begin{aligned}
m_u &= (1.766 \pm .951) \times 10^{-3} GeV \\
m_d &= (3.26 \pm 1.63) \times 10^{-3} GeV \\
m_s &= (62.5 \pm 29.89) \times 10^{-3} GeV \\
\frac{m_u}{m_d} &= (.45 \pm .25) \\
\frac{(m_u + m_d)}{2} &= (2.174 \pm 1.087) \times 10^{-3} GeV \\
\frac{m_s - (m_u + m_d)/2}{m_d - m_u} &= (42.5 \pm 8.5) \\
m_c &= (.576 \pm .069) GeV \\
m_b &= (2.742 \pm .097) GeV \\
m_t &= (166 \pm 5) GeV \\
|V_{us}| &= (.2205 \pm .0035) \\
|V_{ub}| &= (.00315 \pm .00135) \\
|V_{cb}| &= (.039 \pm .003)
\end{aligned}$$

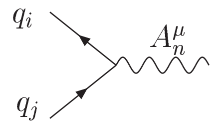
B. Feynman rules

In the following list we summarize the vertices used to carry out the analysis. The derivations including gauge fixing procedure are given in [3].

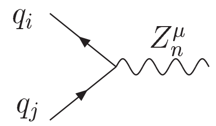
For coupling to the gauge bosons,



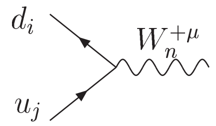
$$ig_s K_n \gamma^\mu T^{(a)}_{\alpha\beta}[U^n_{L\{ij\}}\hat{L} + U^n_{R\{ij\}}\hat{R}]$$



$$ieQ_q K_n \gamma^\mu [U^n_{L\{ij\}}\hat{L} + U^n_{R\{ij\}}\hat{R}]$$



$$\frac{ig_2 K_n}{\cos\theta_W}\gamma^\mu[g^q_L U^n_{L\{ij\}}\hat{L} + g^q_R U^n_{R\{ij\}}\hat{R}]$$



$$\frac{ig_2 K_n}{\sqrt{2}}\gamma^\mu U^n_{L\{ij\}}\hat{L}$$

where $i, j = 1, \ldots, 3$ are the indices for family and α, β stand for color with the chiral projection operators defined as $\hat{R}/\hat{L} = \frac{1}{2}(1 \pm \gamma^5)$ and the standard left-handed/right-handed Z coupling, $g^q_{L/R} = T_3(q) - Q_q \sin^2\theta_W$. The factor K_n lumps together the $\sqrt{2}$ normalization factor for the n-th KK excitation and the gaussian suppression factor,

$$K_n = \begin{cases} \sqrt{2}\exp[-n^2\rho^2/4] & n > 0 \\ 1 & n = 0 \end{cases} \tag{B.1}$$

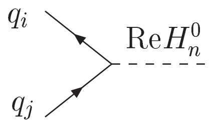
The mixing matrices U^n are given:

$$U^n_{L\{ij\}} = \sum_k (V_L^\dagger)_{\{ik\}} \cos\frac{n y_k^L}{R}(V_L)_{\{kj\}}\,, \tag{B.2}$$

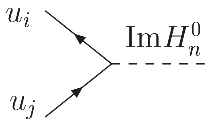
$$U^n_{R\{ij\}} = \sum_k (V_R^\dagger)_{\{ik\}} \cos\frac{n y_k^R}{R}(V_R)_{\{kj\}}\,. \tag{B.3}$$

It is understood that the left/right rotation matrices $V_{L/R}$ are the ones associated with the external fermions. For $n = 0$, the cosine weighting factor reduces to one and the diagonal cosine matrix sandwiched in between also reduces to the three by three identity matrix. Thus, we have $U^0_{L/R\{ij\}} = \delta_{ij}$, which are the SM cases without FCNC and CPV at tree level.

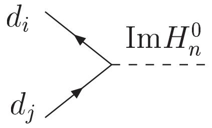
For the Higgs couplings, although complicated, the vertices can also be derived straightforwardly,



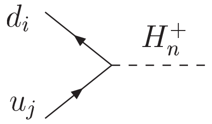
$$\frac{-iK_n}{\sqrt{2}}\left[(h_q^n + h_q^{n\dagger})_{\{ij\}} + \gamma^5(h_q^n - h_q^{n\dagger})_{\{ij\}}\right]$$



$$\frac{-K_n}{\sqrt{2}}\left[(h_u^n - h_u^{n\dagger})_{\{ij\}} + \gamma^5(h_u^n + h_u^{n\dagger})_{\{ij\}}\right]$$



$$\frac{+K_n}{\sqrt{2}}\left[(h_d^n - h_d^{n\dagger})_{\{ij\}} + \gamma^5(h_d^n + h_d^{n\dagger})_{\{ij\}}\right]$$



$$-iK_n\left[\lambda^n_{d\{ij\}}(1+\gamma^5) - (\lambda_u^{n\dagger})_{\{ij\}}(1-\gamma_5)\right]$$

where

$$h^n_{q\{ij\}} = \frac{g_2}{\sqrt{2}M_W}\sum_{k,l=1}^{3}\left[(V_L^q)^\dagger_{\{ik\}}M^q_{\{kl\}}\cos\frac{n\rho(y_k^L + y_l^R)}{2}(V_R^q)_{\{lj\}}\right], \tag{B.4}$$

$$\lambda^n_{d\{ij\}} = \frac{g_2}{\sqrt{2}M_W}\sum_{k,l=1}^{3}\left[(V_L^u)^\dagger_{\{ik\}}M^d_{\{kl\}}\cos\frac{n\rho(y_k^{uL} + y_l^{dR})}{2}(V_R^d)_{\{lj\}}\right], \tag{B.5}$$

$$\lambda^n_{u\{ij\}} = \frac{g_2}{\sqrt{2}M_W}\sum_{k,l=1}^{3}\left[(V_L^d)^\dagger_{\{ik\}}M^u_{\{kl\}}\cos\frac{n\rho(y_k^{dL} + y_l^{uR})}{2}(V_R^u)_{\{lj\}}\right]. \tag{B.6}$$

Again, it can be seen that when $\rho \to 0$ the cosine weighting becomes unity and the vertex couplings reduce to SM cases as expected, namely, $h^n_{q\{ij\}} \to \frac{g_2 m^q}{\sqrt{2}M_W}\delta_{\{ij\}}$, $\lambda^n_{d\{ij\}} \to \frac{g_2 m_j^d}{\sqrt{2}M_W}(V_{CKM}^\dagger)_{\{ij\}}$, and $\lambda^n_{u\{ij\}} \to \frac{g_2 m_i^u}{\sqrt{2}M_W}(V_{CKM}^\dagger)_{\{ij\}}$.

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